Lecture 6: Random Forest

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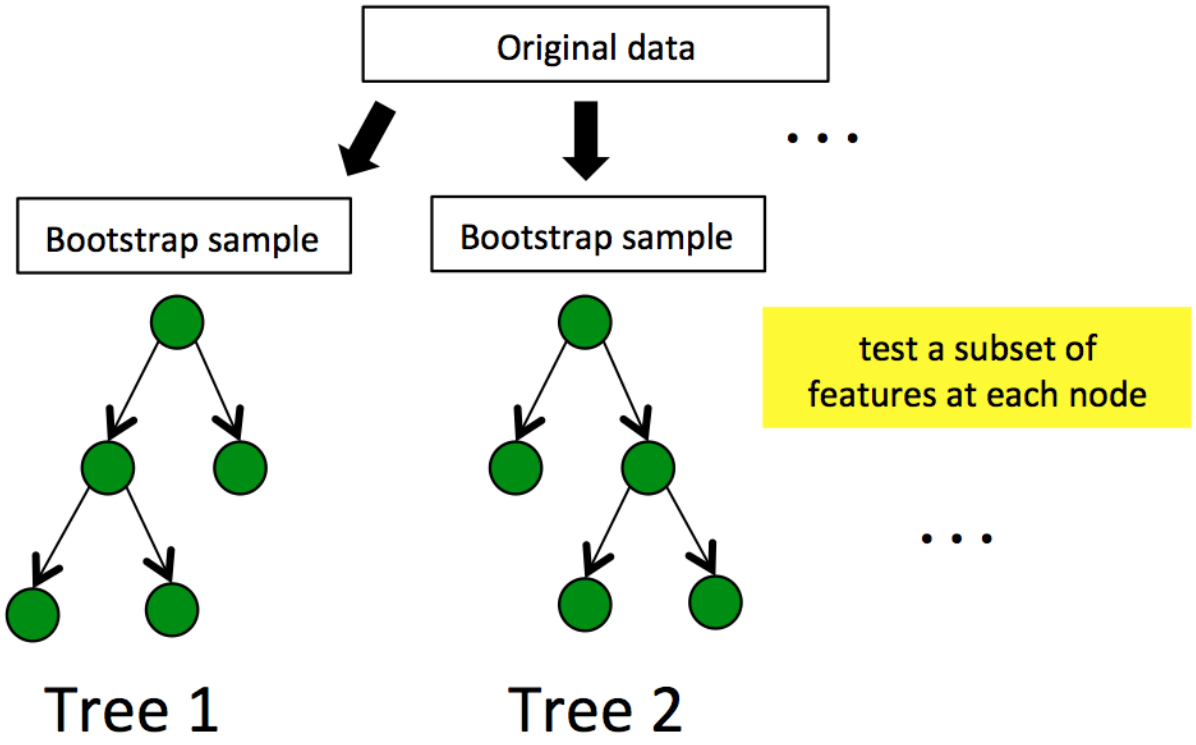
Random forest



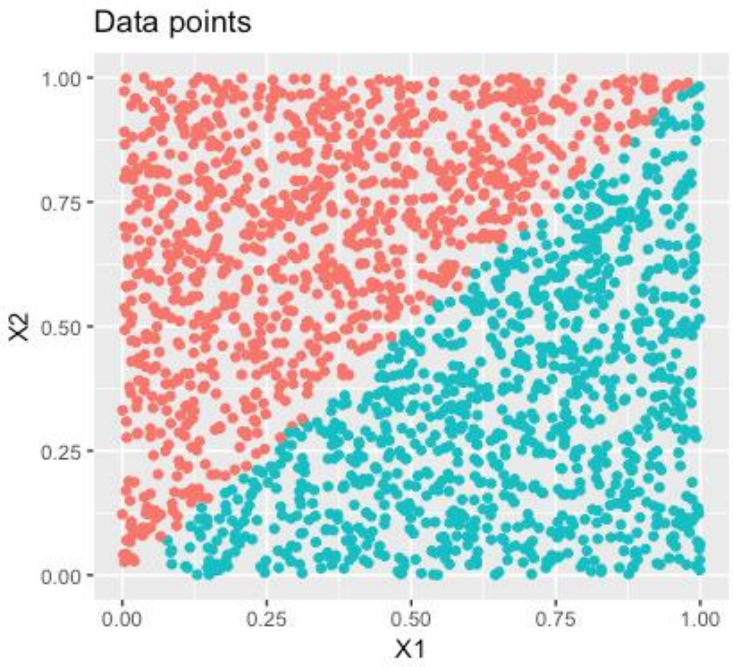
There are two main sources for randomness.

- First, each tree is built on a randomly selected set of samples by applying Bootstrap on the original dataset.
- Second, in building a tree, specifically in splitting a node in the tree, a subset of features is randomly selected to choose the best split.

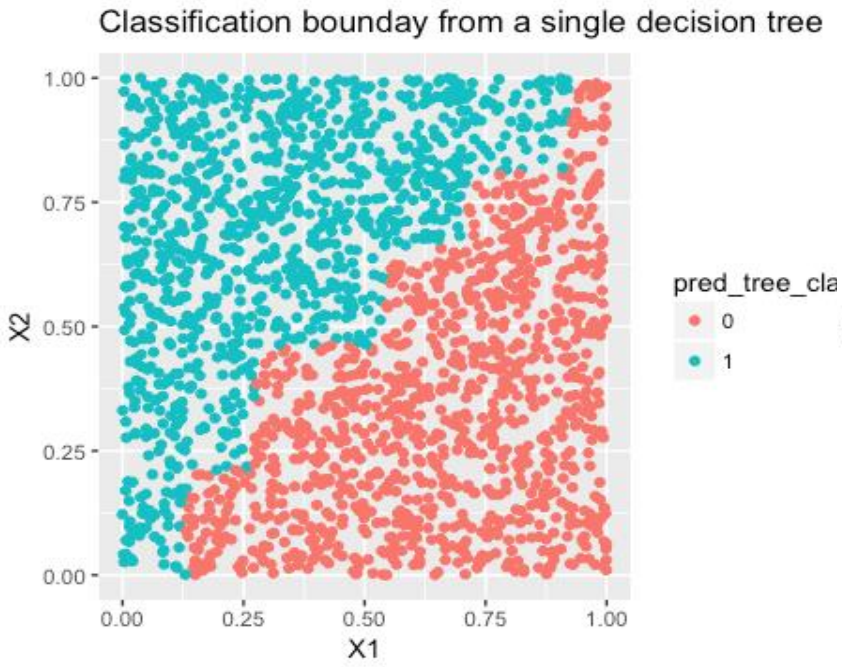
If randomness is troublesome, why we need to ask for it?

Why we need random forest?

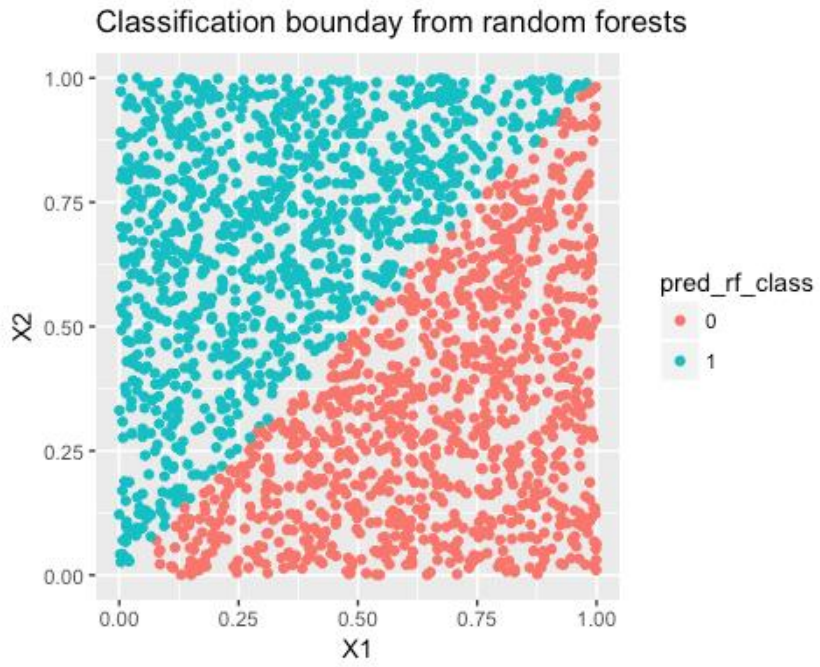
A simulated dataset



Prediction by decision tree



Prediction by random forest



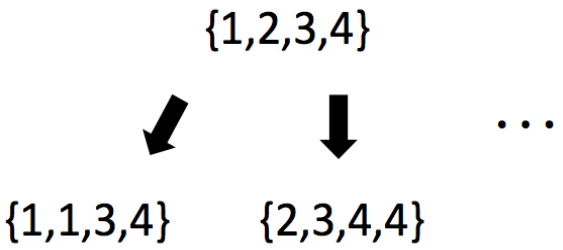
An exemplary data

- Thus, random forest is more of a systematically organized set of heuristics, rather than highly regulated algebraic operations derived from a mathematical characterization.

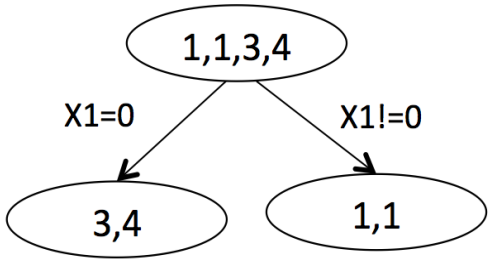
A dataset with 4 instances

ID	X1	X2	Class
1	1	1	C0
2	1	0	C1
3	0	1	C1
4	0	0	C0

Bootstrap the dataset



Build tree on each dataset

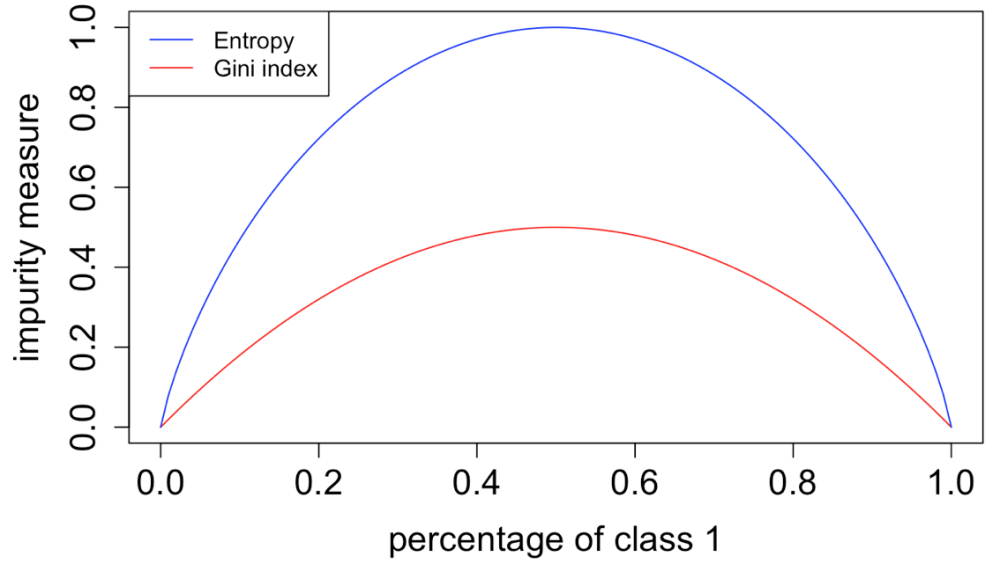


Gini index

- The R package “randomforest” uses the Gini index to measure impurity
- The Gini index is defined as

$$Gini = \sum_{c=1}^{C} p_c(1 - p_c),$$

where C is the number the classes in the dataset, and p_c is the proportion of data instances that come from the class c.

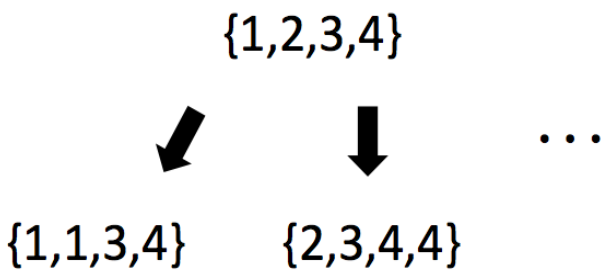


Gini gain

Similar as the information gain, the **Gini gain** can be defined as

$$\nabla\, Gini = Gini - \ w_i Gini_i,$$

where $Gini$ is the Gini index at the node to be split; w_i and $Gini_i$, are the proportion of samples and the Gini index at the i^{th} children node, respectively.

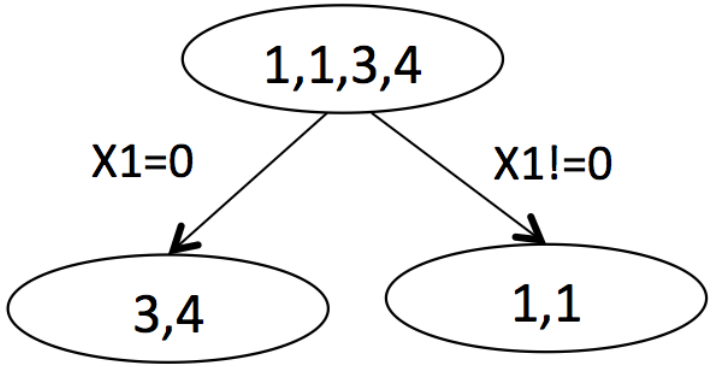


Apply the Gini gain on the exemplary data

- The possible splitting rule candidates include four options: $X_1 = 0$, $X_2 = 0$, $X_1 = 1$ and $X_2 = 1$. Since both variables have two distinct values, both splitting rules $X_1 = 0$ and $X_1 = 1$ will produce the same children nodes, and both splitting rules $X_2 = 0$ and $X_2 = 1$ will produce the same children nodes.
- Therefore, we can reduce the possible splitting rule candidates to two: $X_1 = 0$ and $X_2 = 0$.
- Further, random forest randomly selects variables for splitting a node. In general, for a data set with p predictor variables, $\sqrt{p}$ variables are randomly selected for splitting.
- In our simple example, as there are two variables, we assume that X_1 is randomly selected for splitting the root node.

Apply the Gini gain on the exemplary data – cont'd

- Thus, $X_1 = 0$ is used for splitting the root node



- The Gini index of the root node is calculated as

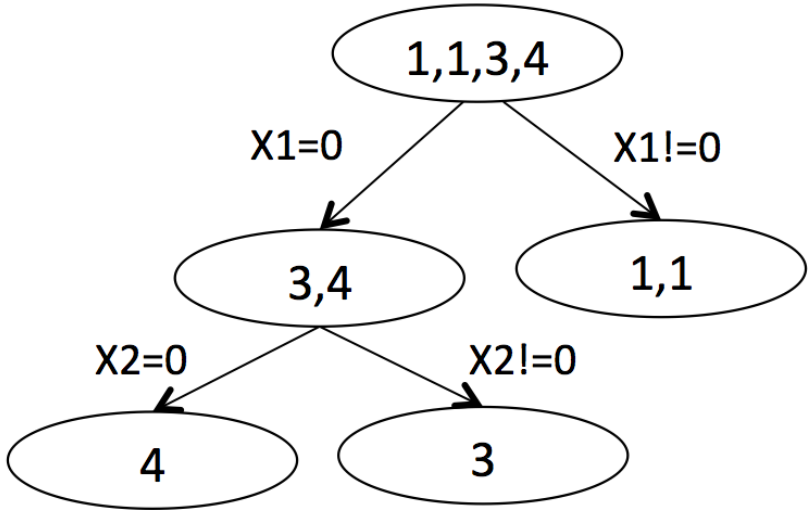
$$\frac{3}{4} * \frac{1}{4} + \frac{1}{4} * \frac{3}{4} = 0.375.$$

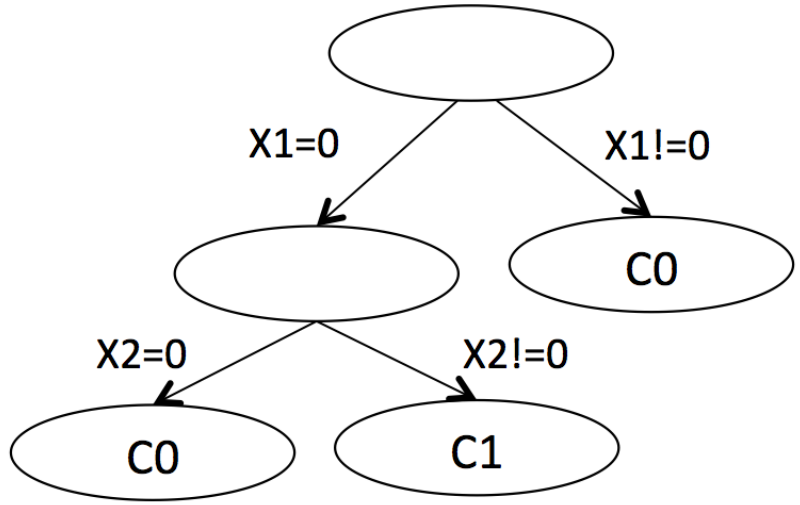
- The Gini gain of this split can be calculated as

$$0.375 - 0.5 * 0 - 0.5 * 0.5 = 0.125.$$

Apply the Gini gain on the exemplary data – cont'd

- Let's continue to grow the tree. Now, at the internal node containing data {3,4}, assume that X_2 is randomly selected. The node can be further split





Why randomness?

- The concept as “weak classifier” is very important in understanding random forest
- Assuming that the trees in random forests are independent, and each tree has an accuracy of 0.6.
- For 100 trees, the probability of random forests to make the right prediction reaches as high as 0.97:

$$\sum_{k=51}^{100} C(n,k) * \ 0.6^k * 0.4^{100-k}.$$

- Note that, the assumption of the independency between the trees in random forests is the key here. This does not hold in reality in a strict sense. However, the randomness added to each tree makes them less correlated.
- This is probably not the answer for why it has to be this way, but it provides an explanation that why it works!

R lab

- Download the markdown code from course website
- Conduct the experiments
- Interpret the results
- Repeat the analysis on other datasets